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# IO-Snap

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## Adding Foreign Imports to IO-Snap Regionalization

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**Abstract.** Prior to this revision, total regional imports have been set to zero prior to supply demand pooling, which balances outputs by comparing total regional supply to total regional demand. Excess supply by commodity is added to the initial estimate of the region's foreign exports, and excess demand is added to imports. Omitting an initial estimate of the regional shares of imports by commodity, however, can result in a substantial underestimate of imports. This document describes the remedy.

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# 1 Common Notation

IO-Snap follows the notational conventions of the commodity-by-industry input-output accounting framework shown in Figure 1, adapted from [United Nations \(1968\)](#).

Figure 1: The Commodity-Industry Framework

	Commodities	Industries	Final Demand	Totals
Commodities		$U$	$e$	$q$
Industries	$V$			$g$
Primary Inputs		$va$		
Totals	$q'$	$g'$		

- $U$  = the *Use* table: row commodities used by column industries
- $V$  = the *Make* table: column commodities produced by row industries
- $e$  = column final demand activities use of row commodities
- $q$  = column vector of total commodity use
- $g$  = column vector of total industry output
- $va$  = column industry payments to row value added components, typically compensation, payments to governments, and gross operating surplus
- $\mathbf{i}$  is a summing vector of appropriate dimension
- $'$  indicates transpose
- $\hat{\phantom{x}}$  indicates diagonalization
- $i, j \Rightarrow$  row and column sector subscripts. When used in combination, subscripts denote a from-to relationship, e.g.,  $z_{i,j}$  denotes a flow from source  $i$  to destination  $j$
- $r, s, N \Rightarrow$  superscripts denoting regions  $r$  and  $s$ , or national variables,  $N$ . When used in combination, superscripts denote origin and destination regions, e.g.,  $z^{rs}$  denotes a flow from region  $r$  to region  $s$

## 2 Reformulating Regional Imports

When we generate regional accounts, we create a scaled-down, regional version of the *Use* and *Make* matrices. We scale the *Use* columns and *Make* rows by the regional shares of national value added. Given the scaled down regional *Make* table, we can

compute regional commodity and industry output,  $q^r$  and  $g^r$ . This allows us to compute regional shares of national industry or commodity output using *Make* row and column sums of the two accounts.

We use regional shares of national variables in a variety of ways. For example, we use regional commodity output shares of national commodity production along with national foreign exports to estimate regional foreign exports by commodity. However, up until October of 2020, instead of providing an initial estimate of regional imports, we set *total* imports to zero prior to the supply-demand balancing algorithm. Although this resulted in an estimate of total – foreign plus interregional – imports. However, omitting the initialization of regional foreign imports had the undesirable consequence of resulting in sub-national accounts that failed to add up to national accounts totals. Specifically, regional total imports were nearly always underestimated.

To remedy this issue, we now provide an initial estimate of foreign imports that will subsequently be adjusted in the supply-demand pooling approach. However, because imports levels are a function of demand rather than supply, we use a regional-shares-of-national variable that will ensure that we meet adding-up constraints.

At the national level, we can define total national (domestic) demand by commodity,  $q_d^N$ , as the sum across the rows of its  $U$  and final demand,  $e$ , excluding the exports and imports columns that are reported in the final demand quadrant of national accounts,  $q_d^N$ . This implies that no imports are re-exported and can introduce some small inaccuracies, but because we have no consistent estimates of both regional and national re-exports, we make no attempt to remedy this potential bias. Summarizing,

$$q_d^N = U^N \mathbf{i} + e^N \mathbf{i} \quad (1)$$

Once we have scaled  $U$  and created a regional counterpart to national total final demands by commodity  $e^r$ , including household consumption,  $PCE^r$ , but again excluding exports (and national imports, as at this point we have no estimate), we can sum across the rows of  $U^r$  and  $e^r$  to generate a vector of total regional demands by commodity, or

$$q_d^r = U^r \mathbf{i} + e^r \mathbf{i} \quad (2)$$

To compute our estimate of regional foreign imports by commodity, we define the regional-shares-of-national variable as

$$mshares^r = (\widehat{q_d^N})^{-1}(q_d^r) \quad (3)$$

Note that  $q_d^N$  will be the same for all regions' imports shares of national for a given year, and that the  $\sum mshares^r \approx \mathbf{i}$ . Strict equality cannot be guaranteed because of the potential for small deviations due mostly to the way in which we estimate regional personal consumption expenditures, which is not by scaling national  $PCE$ , but instead drawing the estimates from published  $PCE$  by state estimates.<sup>1</sup> We make this

<sup>1</sup><https://www.bea.gov/data/income-saving/personal-income-by-state>

decision because we believe that the error introduced is justified by the substantial variation in *PCE* across states is less significant than error that would be introduced by using national averages for state-specific *PCE*. However, regional supply-demand balances will be ensured in the the supply-demand pooling algorithmic step.

Denoting national imports by  $m^N$  and regional imports by  $m^r$ , We can now estimate foreign imports as

$$m^r = \widehat{mshares}^r(m^N) \quad (4)$$

Foreign imports for region  $r$  is now the initial estimate of total imports, replacing what until now was a vector of zeros. Supply-demand pooling then proceeds as before. Supply surpluses are added to the foreign exports vector, and supply deficits are added to foreign imports to generate the total regional exports and total regional imports vectors.

### 3 Defining Key Variables

In this section, we define  $Q$ ,  $D$ , and  $\tilde{D}$ , key variables in our regionalization method. All variables in this section are regional variables, so the  $r$  superscript is omitted. A more complete presentation can be found in [Jackson and Járosi \(2020\)](#).

In [Jackson \(1998\)](#),  $\tilde{D}$  was defined as the *make* matrix standardized by  $s = q + m$ , or  $\tilde{D} = V\hat{s}^{-1}$ . However, further analysis revealed that the original formulation implied that imports would partially satisfy not only domestic demand but also export demand. A formulation in which export demand will be satisfied instead by domestic production within the CxI framework is obtained by recasting the RSP (Regional Supply Percentage) in commodity space. To do so, we define vectors  $m$  and  $ex$  as the regional commodity import and export vectors following the supply-demand pooling adjustments as discussed in the previous section, and let

$$Q = (\mathbf{i}\widehat{V - ex + m})^{-1}(\mathbf{i}V - ex) = (q - \widehat{ex + m})^{-1}(q - ex) \quad (5)$$

where  $V$  is the regional *Make* matrix,  $q$  is a vector of regional commodity output, and

$$D = V\hat{q}^{-1}. \quad (6)$$

Now define  $\tilde{D}$  as

$$\tilde{D} \equiv D\hat{Q}. \quad (7)$$

Substituting equation 5 into equation 7, we also see that

$$\tilde{D} = D(q - \widehat{ex + m})^{-1}(q - ex) = D(\mathbf{i}\widehat{V - ex + m})^{-1}(\mathbf{i}V - ex) \quad (8)$$

## References

- Jackson, R. and Járosi, P. (2020). Consistent regional commodity-by-industry input-output accounts. *WVU Regional Research Institute Working Paper Series, 2020-03*. [https://researchrepository.wvu.edu/rri\\_pubs/215/](https://researchrepository.wvu.edu/rri_pubs/215/).
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